Exam 2
Chapters 14 and 15

Name:_____

_____

Do not write your name on any other page. Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **No calculators are allowed on this exam.**

Show your work!

1. (5 points each)
Find the domain of the following multi-variable functions.

(a) $f(x,y) = \frac{\sqrt{1-x^2-y^2}}{3}$.

(b) $g(x,y,z) = \frac{\ln z}{x^2+y^2}$.

2. Let $z = x^2 \sin xy$ and find the differential dz. (If you don't remember what dz is you can find $\partial z/\partial x$ and $\partial z/\partial y$ for partial credit.) (10 points)

3. Let $z = f(x, y)$ be differentiable and let

$$x = g(t) \quad y = h(t) \quad g(3) = 2 \quad h(3) = 7$$

$$g'(3) = 5 \quad h'(3) = -4 \quad f_x(2, 7) = 6 \quad f_y(2, 7) = -8.$$

Find dz/dt when $t = 3$. (10 points)

4. Let $z = e^{x+2y}$, $x = s/t$, and $y = t/s$. Find $\partial z/\partial s$ and $\partial z/\partial t$. (10 points)

5. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2 - 3y^2 - 9z^2}$$

where T is measured in $°C$ and x, y, z in meters.

(a) Find the rate of change of temperature at the point $P(2, -1, 2)$ in the direction toward the point $Q(3, -3, 3)$. (10 points)

(b) In which direction does the temperature increase the fastest at P? (5 points)

(c) Find the maximum rate of change at P. (5 points)

6. Find the absolute maximum and minimum of $f(x, y) = x^3 - 3x - y^3 + 12y$ on the quadrilateral whose vertices are $(-2, 3), (2, 3), (2, 2),$ and $(-2, -2)$. (15 points)

7. Calculate the double integral (10 points)

$$\iint_R \cos(x + 2y)dA, \quad R = \{(x, y)|0 \leq x \leq \pi, 0 \leq y \leq \pi/2\}.$$

8. Consider the function given by $f(x, y) = xy + 2x - 3y$ and the region D bounded by the circle with center at the origin and radius 3 and the x-axis.

(a) Write the double integral $\iint_D f(x, y) dA$ as a Type I region. (5 points)

(b) Write the double integral $\iint_D f(x, y) dA$ as a Type II region. (5 points)

(c) Evaluate the double integral $\iint_D f(x, y) dA$ in whichever way you wish. (5 points)

Extra Credit

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1. What is your favorite sports team? (15 points)